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LUMPED GEOPOTENTIAL HARMONICS OF ORDER 29, FROM ANALYSIS OF THE ORBIT OF COSMOS 837 ROCKET

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LUMPED GEOPOTENTIAL HARMONICS OF ORDER 29, FROM ANALYSIS OF THE ORBIT OF COSMOS 837 ROCKET

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SUMMARY

The orbit of Cosmos 837 rocket (1976-62E) has been determined at 36 epochs between January and September 1978, using the RAE orbit refinement program PROP6 with about 3000 observations. The inclination was 62.7° and the eccentricity 0.039. The orbital accuracy achieved was between 30 m and 150 m, both radial and crosstrack.

The orbit was near 29:2 resonance in 1978 (exact resonance occurred on May 14) and the values of orbital inclination obtained have been analysed to derive lumped 29th-order geopotential harmonic coefficients, namely:

$$10^9 \bar{C}_{29}^{0,2} = -10 \pm 15 \quad \text{and} \quad 10^9 \bar{S}_{29}^{0,2} = -76 \pm 12 \ .$$

These will be used in future, when enough results at different inclinations have accumulated, to determine individual coefficients of order 29. The values of lumped harmonics obtained from analysis of the values of eccentricity were not well defined, because of the high correlations between them and the errors in removing the very large perturbation (31 km) due to odd zonal harmonics.

Departmental Reference: Space 597

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1 INTRODUCTION

Cosmos 837 rocket, 1976-62E, entered the following orbit on 1976 July 1: inclination 62.75°, perigee height 440 km, apogee height 920 km, period 98.4 min and eccentricity 0.034. Decay is expected about 1984.

The orbit seemed promising for analysis to obtain 29th-order 'lumped' geopotential harmonic coefficients, by virtue of the relatively slow passage through 29:2 resonance with the Earth's gravitational field - when the satellite's track over the Earth's surface repeats every two days while the satellite makes 29 revolutions of the Earth. The orbit has been analysed for the nine-month period 1978 January to September when the effects of 29:2 resonance were significant: exact 29:2 resonance was on 1978 May 14.

The orbit was determined at 36 epochs using the RAE computer program PROP6 [1]. The inclination and eccentricity values, after clearance of perturbations, were fitted by least-squares theoretical curves, using the THROE program[2]. The best fittings gave lumped 29th-order coefficients which are nominally the most accurate so far obtained. These lumped coefficients, with others at different inclinations, will be used to obtain individual 29th-order harmonic coefficients. A few other values have previously been determined[3-5], but several more are required, especially at inclinations near 90°.

2 ORBIT DETERMINATION

2.1 Observations

Over 3100 observations were available for the 36 orbits selected for determination, over the period 1978 January-September. About 15% of these observations were rejected due to not fitting well, to leave a working average of 74 observations per orbit. Three of the orbits had the benefit of highly-accurate Hewitt camera observations.

The largest group of observations used was about 2300 from the US Navy; a further 400 came from British radar, 130 from the kinetheodolite at the South African Astronomical Observatory (SAAO), in the southern hemisphere, 32 from the theodolite in Jokioinen, Finland and nearly 300 observations were from volunteer visual observers, supplied by the Appleton Laboratory at Slough.

In the course of the orbit determinations, it was noticed that several groups of the South African observations were rejected, many with consistent errors of -3 or -4 seconds. Correction of these timing errors led to satisfactory acceptance; subsequently, the SAAO confirmed that such errors had been made in these observations (and possibly others) over a 5-week period at the beginning of 1978.

2.2 The orbits and their accuracy

The 36 computed orbits are given in Table 1, where it can be seen that the sd in inclination, i , varies from 0.0003° to 0.0017°, the rms value being 0.0008°; by comparison, the rms for the three Hewitt camera runs is 0.0005°. For eccentricity, e , the sd varies from 4×10^{-6} to 19×10^{-6}. The best sd values of i and e are equivalent to 30 m in position. For the right ascension of the node, Ω , the average sd is 0.0009°, equivalent to 100 m.

Table 1

Orbital parameters for Cosmos 837 rocket, with standard deviations

	MJD	Date 1978	a	e	i	Ω	ω	M_0	M_1	M_2	ε	D	N
1	43514.0	Jan 6	7056.6206	0.038256	62.7406	15.7015	2.181	14.924	5273.0814	0.02374	0.52	8.9	61
			2	14	11	9	10	10	3	10			
2	522.0	14	7056.3017	0.038355	62.7417	349.9948	3.510	81.071	5273.4390	0.01961	0.44	8.9	63
			2	8	8	8	8	8	2	8			
3	532.0	24	7055.9888	0.038501	62.7420	317.8549	5.184	257.305	5273.7898	0.01666	0.56	8.6	76
			3	7	8	10	10	10	3	10			
4*	540.0	Feb 1	7055.6339	0.038559	62.7423	292.1450	6.504	329.042	5274.1879	0.03183	0.50	7.6	65
			3	8	3	8	10	10	3	16			
5	548.0	9	7055.2032	0.038676	62.7386	266.4234	7.818	44.588	5274.6707	0.02684	0.49	7.6	64
			3	11	11	10	10	10	3	15			
6	555.0	16	7054.9588	0.038812	62.7382	243.9120	8.947	248.384	5274.9447	0.01785	0.61	6.9	60
			3	6	6	6	7	7	3	14			
7	564.0	25	7054.7220	0.038966	62.7377	214.9628	10.464	204.080	5275.2103	0.01677	0.64	9.3	48
			3	19	17	15	17	16	4	15			
8	574.0	Mar 7	7054.3304	0.039081	62.7388	182.7971	12.042	38.375	5275.6499	0.02384	0.53	8.9	61
			3	13	12	10	11	11	3	14			
9	584.0	17	7053.8633	0.039178	62.7431	150.6240	13.683	237.579	5276.1743	0.02422	0.60	9.1	73
			2	5	8	11	8	8	3	6			
10	594.0	27	7053.4390	0.039300	62.7375	118.4402	15.405	81.595	5276.6500	0.02807	0.77	9.4	59
			3	8	13	11	15	14	3	13			
11	604.0	Apr 6	7052.7617	0.039338	62.7345	86.2450	16.990	291.491	5277.4101	0.05014	0.54	5.9	69
			4	9	8	10	10	9	4	21			
12*	611.0	13	7052.0397	0.039385	62.7365	63.7034	18.072	156.130	5278.2209	0.06162	0.67	5.1	66
			4	5	5	7	8	8	4	20			
13	616.0	18	7051.5292	0.039357	62.7385	47.5931	18.934	268.728	5278.7942	0.05279	0.49	5.7	63
			6	11	9	9	12	11	7	28			
14	624.0	26	7050.8419	0.039424	62.7402	21.8188	20.228	22.365	5279.5664	0.04268	0.47	8.9	76
			2	11	9	8	11	10	2	11			
15	633.0	May 5	7050.1571	0.039559	62.7371	352.8086	21.655	22.026	5280.3355	0.04120	0.58	7.0	51
			6	17	14	13	15	15	7	26			
16	641.0	13	7049.6910	0.039619	62.7373	327.0119	22.885	147.009	5280.8593	0.02927	0.53	7.0	74
			2	7	6	5	6	6	3	14			
17	648.0	20	7049.3633	0.039675	62.7356	304.4364	24.008	34.427	5281.2275	0.02429	0.38	7.0	85
			1	8	7	6	8	7	1	8			
18	655.0	27	7049.0539	0.039729	62.7389	281.8564	25.127	284.325	5281.5755	0.02223	0.34	7.3	[illegible]
			1	4	5	5	5	5	2	7			
19	665.0	Jun 6	7048.5968	0.039820	62.7417	249.5988	26.683	182.741	5282.0896	0.02080	0.50	7.0	57
			3	13	11	9	13	12	4	16			
20	673.0	14	7048.3420	0.039880	62.7421	223.7865	27.950	320.727	5282.3761	0.01796	0.36	7.7	73
			1	8	6	7	8	8	1	6			
21	681.0	22	7047.9572	0.039976	62.7381	197.9700	29.175	101.303	5282.8085	0.03667	0.58	5.9	82
			5	12	8	9	14	13	5	18			
22	688.0	29	7047.3403	0.040027	62.7362	175.3735	30.235	3.401	5283.5021	0.05260	0.32	5.9	76
			1	9	5	4	9	9	2	9			
23	695.0	Jul 6	7046.7772	0.040034	62.7372	152.7693	31.344	270.317	5284.1356	0.04248	0.63	6.9	81
			2	15	9	8	17	16	3	13			
24*	701.0	12	7046.3310	0.040034	62.7387	133.3921	32.197	296.754	5284.6377	0.04846	0.60	5.6	70
			3	14	6	12	16	15	4	21			
25	707.0	18	7045.7703	0.040048	62.7407	114.0078	33.181	326.465	5285.2688	0.05581	0.64	6.3	70
			4	16	10	16	19	18	5	21			
26	714.0	25	7045.1294	0.040104	62.7388	91.3853	34.349	246.029	5285.9900	0.04316	0.53	4.6	75
			4	11	8	9	13	12	4	29			
27	718.0	29	7044.8866	0.040138	62.7384	78.4571	34.980	150.641	5286.2633	0.02764	0.40	4.9	84
			2	7	5	6	9	9	3	18			
28	725.0	Aug 5	7044.5810	0.040210	62.7379	55.8278	35.992	75.884	5286.6074	0.02497	0.49	6.9	85
			2	5	6	5	5	5	2	11			
29	732.0	12	7044.3127	0.040245	62.7363	33.1901	37.117	3.319	5286.9093	0.01973	0.47	8.0	84
			2	9	6	9	11	11	2	9			
30	739.0	19	7044.1111	0.040323	62.7377	10.5516	38.192	292.658	5287.1365	0.01304	0.61	6.6	81
			3	12	7	10	16	15	3	16			
31	746.0	26	7043.9807	0.040411	62.7402	347.9142	39.285	223.253	5287.2834	0.01167	0.63	6.2	84
			3	11	8	11	16	15	3	16			
32	754.0	Sep 3	7043.7319	0.040486	62.7414	322.0416	40.489	42.709	5287.5638	0.01967	0.47	7.9	89
			2	8	6	9	12	11	2	9			
33	761.0	10	704[illegible].4281	0.040532	62.7380	299.3963	41.587	336.842	5287.9056	0.02966	0.45	5.9	83
			2	9	6	7	13	12	2	15			
34	767.0	16	7043.0906	0.040578	62.7338	279.9835	42.526	25.473	5288.2855	0.03315	0.47	6.0	90
			2	9	6	6	12	11	2	13			
35	773.0	22	7042.6663	0.040587	62.7334	260.5642	43.428	76.614	5288.7634	0.05089	0.47	5.0	76
			2	8	7	8	13	12	3	21			
36	779.0	28	7041.9341	0.040579	62.7344	241.1395	44.386	131.473	5289.5885	0.08715	0.55	5.6	96
			3	10	7	7	15	14	3	15			

Key:
- MJD Modified Julian Day
- a semi major axis (km)
- e eccentricity
- i inclination (deg)
- Ω right ascension of node (deg)
- ω argument of perigee (deg)
- * orbits with Hewitt camera observations
- M_0 mean anomaly at epoch (deg)
- M_1 mean motion, n (deg/day)
- M_2 third coefficient in polynomial for mean anomaly
- ε measure of fit
- D time coverage of observations (days)
- N number of observations used

2.3 Motion of perigee

Since the inclination is close to 63°, the perigee moves very slowly, at less than 0.2 deg/day, and the argument of perigee ω increases from 2° to 44° in the 265 days between first and last epochs, as shown in Fig 1. The odd zonal harmonics in the geopotential have a great effect on eccentricity when $i = 62.74^{\circ}$: the amplitude of the oscillation is nearly 50 km, so a decrease in perigee distance of over 30 km is to be expected as ω increases from 2° to 44°. In fact, $a(1 - e)$ decreases from 6787 km on the first orbit to 6756 km on the last. This corresponds to a decrease in perigee height over a spherical Earth from 409 km initially to 378 km at the end, Fig 1. Over an oblate Earth, the corresponding values are 409 km and 386 km.

2.4 Observational accuracy

The residuals of the observations on the first 32 orbits are summarized in Table 2 for stations with 5 or more observations accepted. The residual of an observation is a combination of the observational error and any error in the orbit, and the value given in the Table, the rms, produces a bias towards the larger values, which for visual observations relative to the stars are usually observations made in poor conditions of seeing. For these reasons the capability of visual observers in good conditions is usually reckoned to be about half the rms residual. For the US Navy station 29, the angular residuals are geocentric, and need to be multiplied by a factor of about 5 for comparison with the other (topocentric) observations. All observers with at least one observation accepted have been sent copies of their residuals.

Table 2

Residuals for observing stations with more than 5 observations accepted

Station		Number of observations accepted	Rms residuals			
			Range km	Minutes of arc		
				RA	Dec	Total
1	US Navy	187		1.7	1.9	2.6
2	US Navy	130		2.1	2.2	3.0
3	US Navy	134		2.0	1.9	2.8
4	US Navy	141		1.6	2.0	2.6
5	US Navy	161		1.9	1.7	2.6
6	US Navy	175		1.6	1.8	2.4
29	US Navy	625	0.5	0.3*	0.4*	
414+	Cape Town	28		2.3	2.3	3.2
2122+	Malvern 5	13		1.6	1.2	2.1
2125+	Street	7		2.9	1.9	3.5
2155+	Bahrein 2	8		2.8	4.4	5.2
2265+	Farnham	6		2.0	1.3	2.4
2303	Malvern Hewitt camera	9		0.02	0.02	0.03
2414+	Bournemouth	59		3.6	3.5	5.0
2420+	Willowbrae	28		1.9	2.1	2.8
2577	Cape kinetheodolite	75		0.8	0.7	1.1
4168+	Vries	11		4.0	3.8	5.5
6702+	Jokioinen	23		3.0	3.6	4.7

* Geocentric
\+ Visual stations

3 THE 29:2 RESONANCE

3.1 Analysis of inclination, i

The theory for 29:2 resonance is detailed in Refs 3-5, where all the parameters used here are defined. The first term of the inclination equation is

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{30}(29 - 2\cos i)\bar{F}_{30,29,14}\left(\bar{S}_{29}^{0,2}\sin\Phi + \bar{C}_{29}^{0,2}\cos\Phi\right) , \tag{1}$$

where $\Phi = 2(\omega + M) + 29(\Omega - \nu)$

is the resonance angle, ν being the sidereal angle. The values of Φ and $\dot{\Phi}$ are given in Fig 2: at exact resonance $\dot{\Phi} = 0$. The use of two extra terms in (1) - taking $(\gamma,q) = (1,0)$, $(1,1)$ and $(1,-1)$ in the notation of Ref 4 - gave indeterminate results, very probably because the $(\Phi \pm \omega)$ terms interfered with the Φ terms as a result of the very slow variation of ω. The fitting of i was therefore made with equation (1), in integrated form, using the THROE computer program[2].

Before being fitted by THROE, the 36 values of inclination from Table 1 were first cleared of: zonal harmonic and lunisolar perturbations (combined maximum value being 0.0070°), using the PROD program[6] with numerical integration at daily intervals; tesseral harmonic perturbations, determined by PROP (maximum 0.0016°); and atmospheric-rotation perturbations (maximum 0.0018°) determined within THROE, using an atmospheric rotation rate Λ of 1.0, which gave a better fitting than the other alternative tried, $\Lambda = 0.9$. Earth and ocean tide effects were not taken into account, and, in recognition of this, the sd of one value of inclination, on orbit 4, was degraded from 0.0003° to 0.0005°. A density scale height H of 60 km was used, appropriate to a mean height of 430 km (0.75 H above mean perigee height).

The 36 modified inclination values were then fitted by the integrated form of equation (1), using THROE. In the first fitting the measure of fit ε was 1.5, so the nine worst-fitting values were degraded successively, two by a factor of 3 and seven by a factor of 2. Fig 3 shows the final fitting, and the values with their original standard deviations, the nine degraded values being marked by horizontal bars. The values of the lumped coefficients finally obtained were:

$$10^{9}\bar{C}_{29}^{0,2} = -10 \pm 15 , \qquad 10^{9}\bar{S}_{29}^{0,2} = -76 \pm 12 , \tag{2}$$

with $\varepsilon = 0.88$. These were within 1 sd of the values obtained on the first fitting. Fig 3 shows that the overall effect of the resonance was to increase the inclination by about 0.003°.

For 1971-62E the numerical expression for $\bar{C}_{29}^{0,2}$ in terms of the individual coefficients, as obtained from the RAE computer program PROF, is

$$\bar{C}_{29}^{0,2} = \bar{C}_{30,29} - 2.35\bar{C}_{32,29} + 1.48\bar{C}_{34,29} + 0.82\bar{C}_{36,29} - 0.70\bar{C}_{38,29} - 0.66\bar{C}_{40,29} + \ldots, \tag{3}$$

with the same equation for S, on replacing C by S throughout.

3.2 Analysis of eccentricity, e

In attempting to analyse the variations in eccentricity, the 36 values of e from Table 1 were first corrected for lunisolar perturbations, using PROD, and then the values of M_2 were modified, being replaced by $\frac{(M_1)_{n+1} - (M_1)_n}{2(t_{n+1} - t_n)}$, where $(M_1)_n$ is the value of M_1 on the nth orbit, at epoch t_n. This allows for the integrated effect of drag between successive epochs[4], as required by THROE. (This correction is not significant in fitting i.) In the THROE runs the scale height H has to be taken at a height 1.5H above perigee, and a value of 65 km was used.

The appropriate equation for fitting the variation in e due to resonance is[4]

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{29}\left[-16\bar{F}_{29,29,14}\left\{\bar{C}_{29}^{1,1}\sin(\Phi-\omega) - \bar{S}_{29}^{1,1}\cos(\Phi-\omega)\right\}\right.$$

$$\left. + 12\bar{F}_{29,29,13}\left\{\bar{C}_{29}^{-1,3}\sin(\Phi+\omega) - \bar{S}_{29}^{-1,3}\cos(\Phi+\omega)\right\}\right], \qquad (4)$$

taking $(\gamma,q) = (1,1)$ and $(1,-1)$. Accordingly, the values of e, after removal of air-drag and zonal-harmonic perturbations within THROE, were fitted by THROE using equation (4) in integrated form. Unfortunately the values obtained for the lumped coefficients $(C,S)_{29}^{1,1}$ and $(C,S)_{29}^{-1,3}$ were indeterminate, probably as a result of being highly correlated because ω varies so slowly. Also the value of ε was high (3.0), and, even after five values of e had been degraded in accuracy, thus reducing ε to 2.1, the values of the lumped harmonics were still all less than twice their sd.

The first possible escape route from the 'correlation trap' is to ask whether one pair of terms in (4) is small and can be ignored. For 1976-62E, the PROF computer program gives

$$\left.\begin{aligned}\bar{C}_{29}^{1,1} &= \bar{C}_{29,29} - 6.14\bar{C}_{31,29} + 10.80\bar{C}_{33,29} - 4.69\bar{C}_{35,29} - 5.08\bar{C}_{37,29} + \ldots \\ \bar{C}_{29}^{-1,3} &= \bar{C}_{29,29} - 4.38\bar{C}_{31,29} + 4.37\bar{C}_{33,29} + 1.18\bar{C}_{35,29} - 2.41\bar{C}_{37,29} - \ldots\end{aligned}\right\} \qquad (5)$$

and similarly for S. Equations (5) indicate that the most probable value for the ratio $\bar{C}_{29}^{-1,3}\big/\bar{C}_{29}^{1,1}$ is about ½, and similarly for S. Since the multiplying factors outside the two curly brackets in equation (4) have numerical values of 0.44 and 0.76 respectively, the most probable situation is that the $\bar{C}_{29}^{1,1}$ terms are of about the same magnitude as the $\bar{C}_{29}^{-1,3}$ terms in (4), *so that neither can legitimately be ignored.*

Another escape route might be found by taking the magnitudes of the two terms as equal. Since they are opposite in sign, the equation (4) would reduce to the form $K(\bar{C}_{29}^{1,1}\cos\Phi - \bar{S}_{29}^{1,1}\sin\Phi)\sin\omega$, where K is constant. So if ω increased from, say,

070

to 110°, it would be possible to take a constant value of sin ω and fit a $(\gamma,q) = (1,0)$ variation. But unfortunately sin ω increases steadily from 0 to 0.7 and cannot be taken as constant. Thus ω is too nearly constant to allow separation of the effects of the lumped coefficients, but not constant enough to allow useful simplifications.

Even if one of these escape routes had been open, however, there would still be another obstacle to face: the amplitude of the oscillation in perigee height is very large for 1976-62E (about 46 km), and even the latest set of odd zonal harmonics[7] may well have uncertainties of up to 2 km at this inclination. Since the change due to resonance is likely to be less than 1 km, it is not possible to remove the odd zonal harmonic perturbation with adequate accuracy.

This difficulty, together with the correlation between the lumped coefficients, prevents a satisfactory solution. But the values of eccentricity still need to be allotted some fitted curve for future comparison; so it was decided to adjust the values of the odd zonal harmonics by altering J_7 so that (a) the value of ε was near minimal and (b) the values of the lumped harmonics were consistent with the maximum credible values, found by inserting $(\bar{C},\bar{S})_{\ell,29} = 10^{-5}/\ell^2$ in (5) and taking the sum of the numerical values of the terms. (The individual coefficients are generally less than $10^{-5}/\ell^2$ - see, for example, Ref 8 - and they will rarely all be of the same sign.) This gives $\max.(\bar{C},\bar{S})_{29}^{1,1} \simeq 300 \times 10^{-9}$ and $\max.(\bar{C},\bar{S})_{29}^{-1,3} \simeq 150 \times 10^{-9}$. The best solution, which gave values within 1 sd of these limits and had a reasonably low ε, namely $\varepsilon = 1.6$, was obtained by changing J_7 from the standard value in THROE, -326×10^{-9}, to -306×10^{-9}. The resulting values of lumped harmonics were:

$$\left.\begin{aligned} 10^9\bar{C}_{29}^{1,1} &= 340 \pm 750 \qquad & 10^9\bar{S}_{29}^{1,1} &= -370 \pm 820 \\ 10^9\bar{C}_{29}^{-1,3} &= -320 \pm 430 \qquad & 10^9\bar{S}_{29}^{-1,3} &= 480 \pm 430 \,. \end{aligned}\right\} \tag{6}$$

Fig 4 shows the values of e, with their original sd, and the fitted curve. In the fitting, three of the standard deviations were relaxed by a factor of 3, and nine by a factor of 2. These twelve values are marked by horizontal bars. The fitting is fairly satisfactory, and it is probable that determinate values for the lumped coefficients would have emerged if they had not been so strongly correlated and of similar magnitude.

Since the variation of perigee height with time (Fig 1) is near-linear, another way of removing any error in the calculated amplitude of the odd zonal harmonic perturbation would be to include a linear term in the fitting. This was tried, but the numerical coefficient of the linear term was less than its sd, and the general appearance of the fitting was not appreciably altered.

Finally, the first five values of e, which seem rather low, were omitted, and a fitting with only the second pair of lumped coefficients was made. The resulting values, with $J_7 = -296 \times 10^{-9}$, were:

$$10^9 \bar{C}_{29}^{-1,3} = -462 \pm 93 \qquad 10^9 \bar{S}_{29}^{-1,3} = 302 \pm 46 \quad , \tag{7}$$

with $\varepsilon = 1.39$. The standard deviations in (7) are much lower than in (6), but cannot be accepted as realistic because a change of J_7 to -276×10^{-9} changes the two values to -328 and -49 respectively.

The most useful information to emerge is that, in all the 21 fittings attempted, $\bar{C}_{29}^{-1,3}$ was always strongly negative. Since its numerical value seems unlikely to exceed 150×10^{-9}, the results point towards a value somewhere near -150×10^{-9}.

Although the present analysis of e has not yielded good values of lumped harmonics, this is mainly because of the lack of knowledge of values of the odd zonal harmonics. When these are better established, a successful analysis may be possible, since the orbital data are basically accurate enough.

4 COMPARISON WITH GEM 10B

The only comprehensive gravity model that goes to order and degree 36 is the Goddard Earth Model 10B [9] and recent tests with accurate resonant orbits [10] indicate that the 15th-order terms in GEM 10B are probably accurate to 3×10^{-9}. The terms of order 29 and degree 30-36 are likely to be less accurate, and an error of 5×10^{-9} may be tentatively assigned. (The use of GEM 10C, which is the same as GEM 10B to order and degree 36 but goes to degree 180, is not very useful here, because the terms of degree >36 do not greatly influence the lumped coefficients.)

Assuming an error of 5×10^{-9} and ignoring the terms of degree 38,40,..., GEM 10B gives

$$10^9 \bar{C}_{29}^{0,2} = -8 \pm 15 \qquad 10^9 \bar{S}_{29}^{0,2} = -11 \pm 15 \quad . \tag{8}$$

Comparison with the values (2) from 1976-62E shows good agreement for C and disagreement for S. The high negative value of $\bar{S}_{29}^{0,2}$ obtained from 1976-62E derives directly from the main increase in inclination between MJD 43615 and 43675 (see Fig 3), where $270^\circ < \Phi < 320^\circ$ (see Fig 2) and $\sin \Phi$ in equation (1) therefore has a value between -0.64 and -1.0. This 'main increase' in inclination seems securely based, because it is so strongly confirmed by all the outlying points in Fig 3. So the value of $\bar{S}_{29}^{0,2}$ from equations (2) seems preferable to that from equations (8).

The value of $\bar{C}_{29}^{-1,3}$ from GEM 10B is not likely to be very realistic, because of the neglect of terms of order 37,39,...; but the $\ell = 33$ term in GEM 10B gives a contribution of -56 ± 20 to $10^9 \bar{C}_{29}^{-1,3}$, which is consistent with the strongly negative value indicated by 1976-62E.

5 CONCLUSIONS

The orbit of Cosmos 837 rocket has been determined at 36 epochs spread throughout the first nine months of 1978, when the effects of 29:2 resonance were being felt. About 3100 observations were used, of which about 757 were supplied by the US Navy.

The 36 orbits obtained are given in Table 1 and show that the sd in inclination varies between 0.0003° and 0.0017° while the sd in eccentricity varies from 4×10^{-6} to 19×10^{-6}. The best standard deviations are each equivalent to 30 m in position.

The 36 values of inclination (cleared of zonal-harmonic, $J_{2,2}$, air-drag and lunisolar perturbations) were fitted with a least-squares theoretical curve to give the following values of lumped 29th-order coefficients:

$$10^9 \bar{C}_{29}^{0,2} = -10 \pm 15 \qquad 10^9 \bar{S}_{29}^{0,2} = -76 \pm 12 \ .$$

The analysis of eccentricity was not so successful because of (a) the difficulty of accurately removing the very large perturbation (31 km) due to odd zonal harmonics, and (b) the interference between the two pairs of lumped harmonics, caused by the very slow variation of the argument of perigee at this inclination (62.74°). However, there is a strong indication that $\bar{C}_{29}^{-1,3}$ is strongly negative, of order -150×10^{-9}.

The values of lumped harmonics from 1976-62E - which appear to be the best so far obtained at 29:2 resonance - will be used in future determinations of individual coefficients, when results for a variety of inclinations are available.

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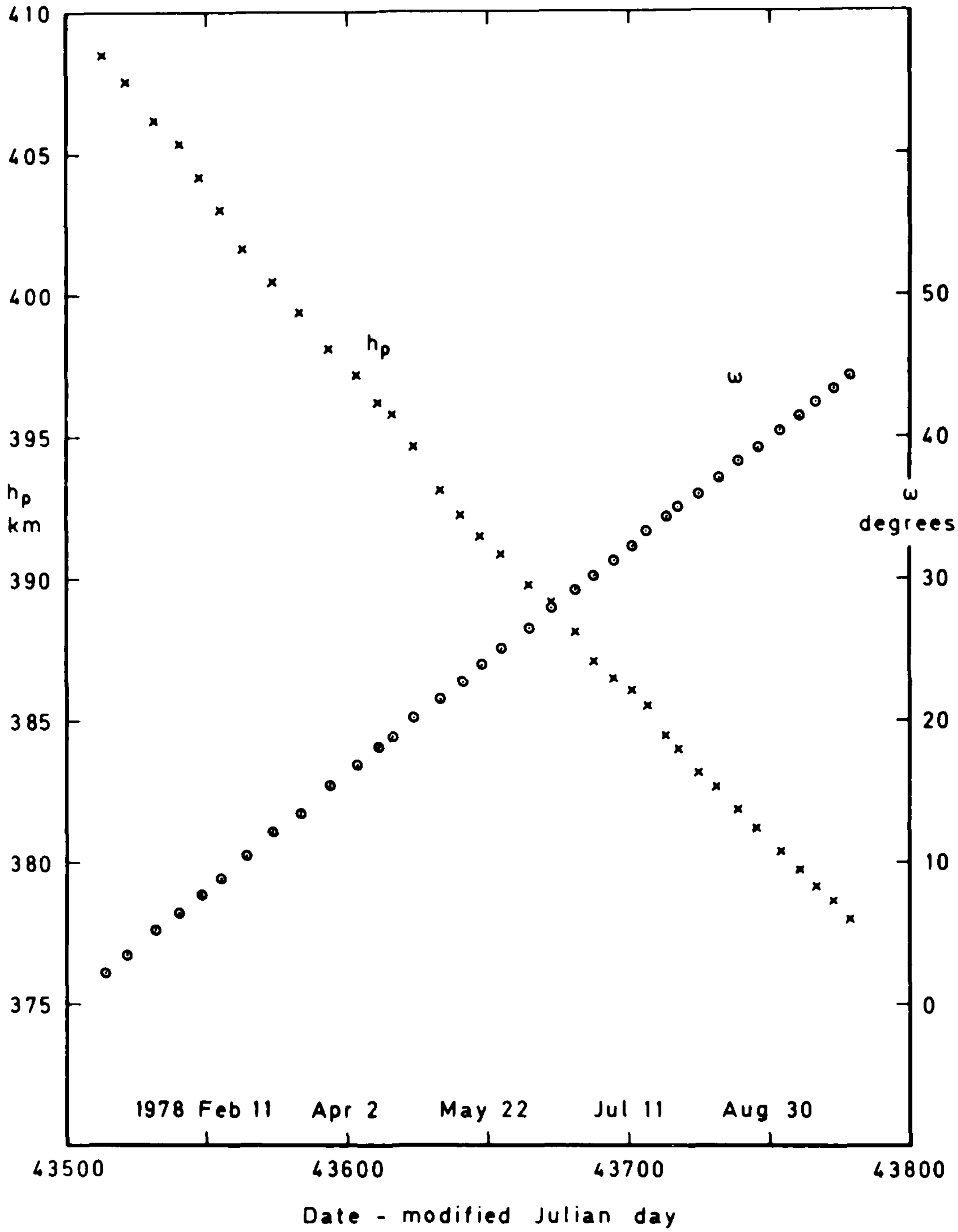

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Fig 1 Perigee height h_p over spherical Earth and argument of perigee, ω

Fig 2

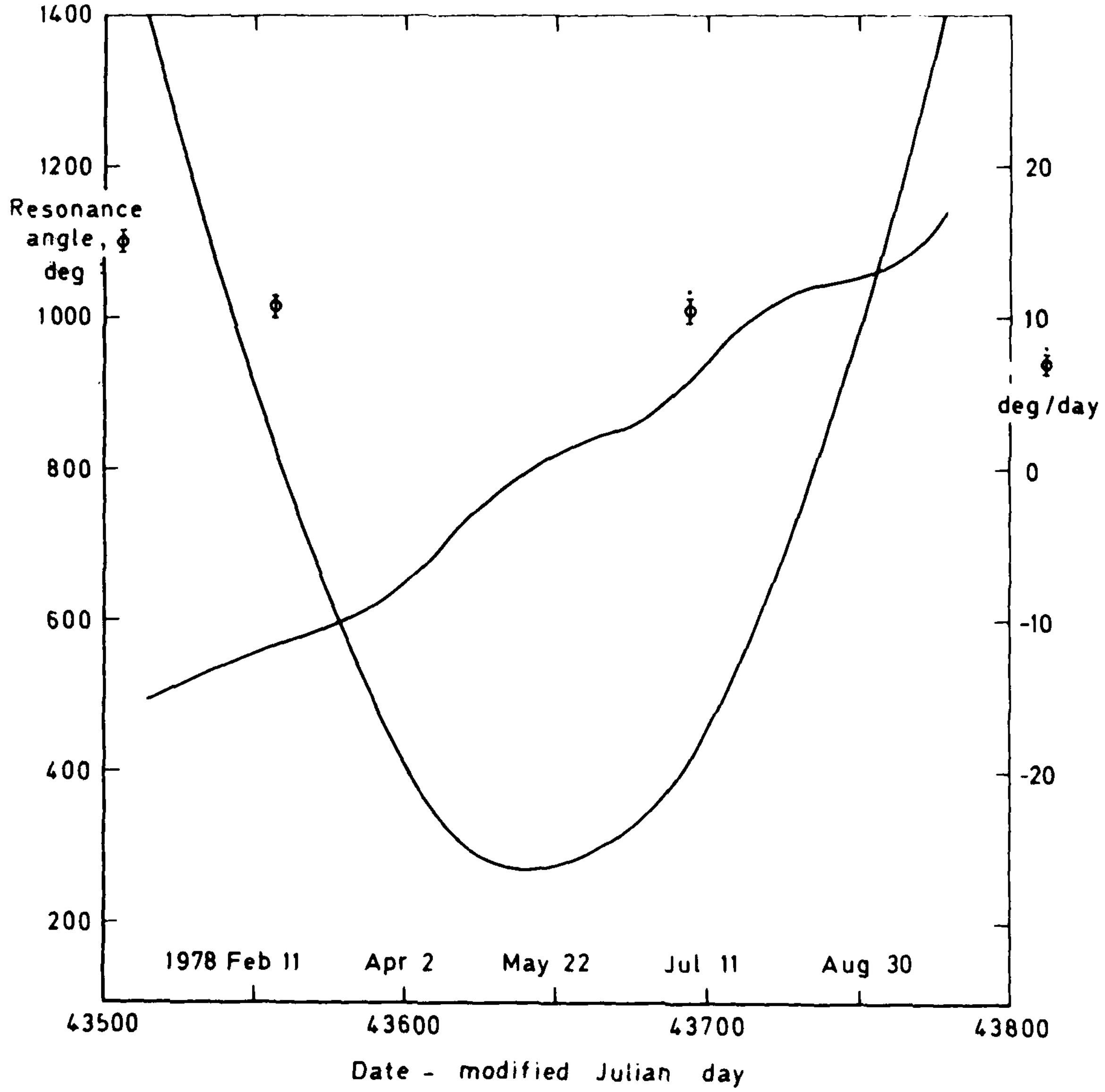

Fig 2 Variation of Φ and $\dot{\Phi}$

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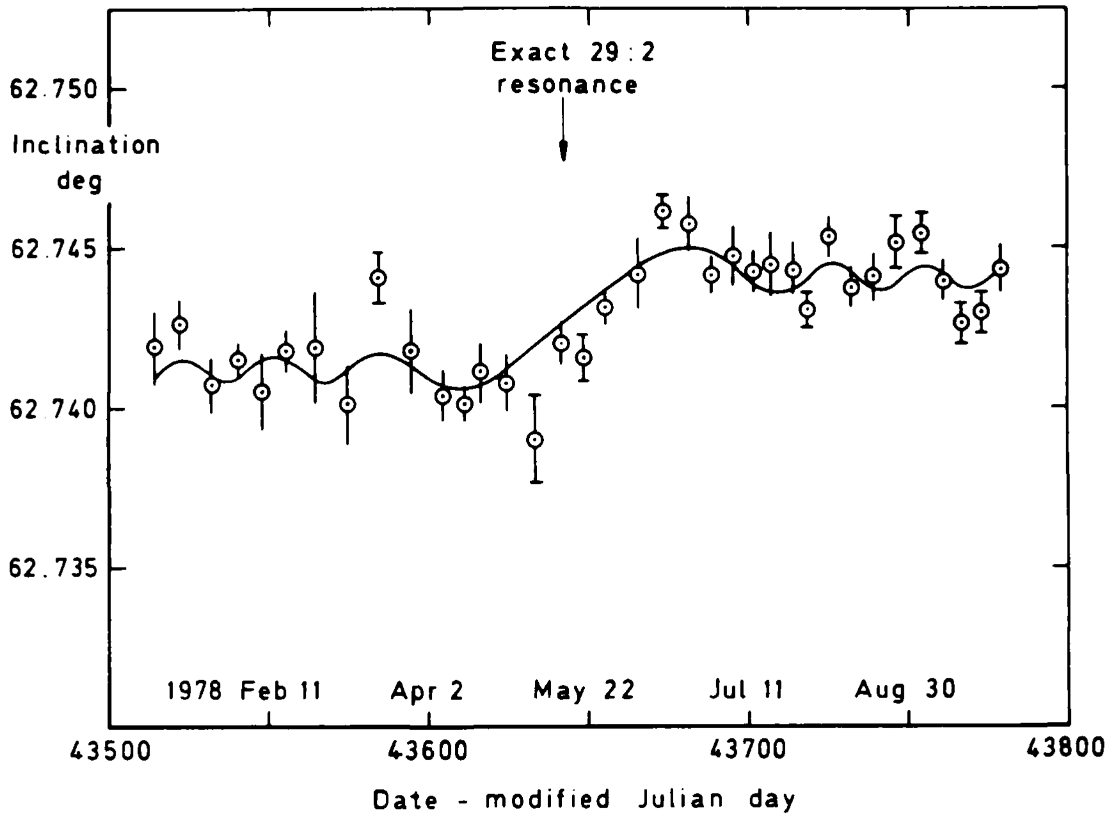

Fig 3 Inclination values near 29:2 resonance, with fitted theoretical curve

Fig 4

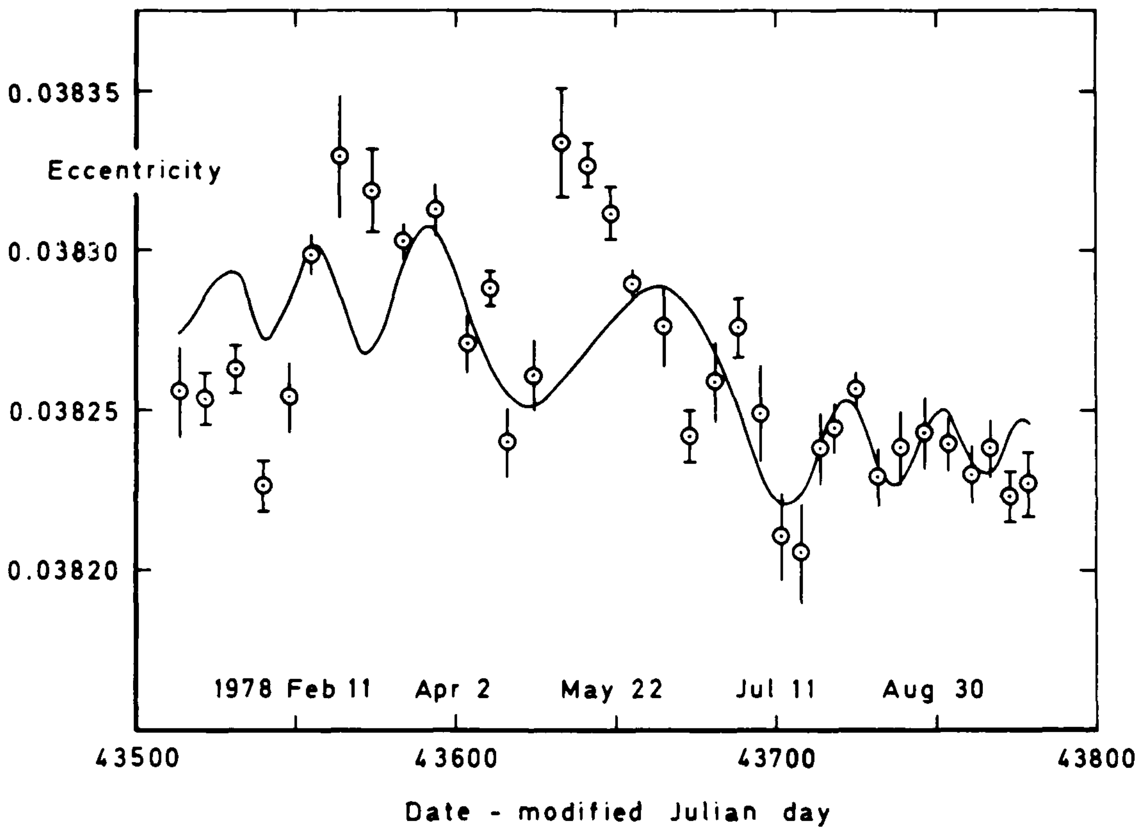

Fig 4 Eccentricity values near 29:2 resonance, with fitted theoretical curve

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17. Abstract

The orbit of Cosmos 837 rocket (1976-62E) has been determined at 36 epochs between January and September 1978, using the RAE orbit refinement program PROP6 with about 3000 observations. The inclination was 62.7° and the eccentricity 0.039. The orbital accuracy achieved was between 30 m and 150 m, both radial and crosstrack.

The orbit was near 29:2 resonance in 1978 (exact resonance occurred on May 14) and the values of orbital inclination obtained have been analysed to derive lumped 29th-order geopotential harmonic coefficients, namely:

$$10^9 \bar{C}_{29}^{0,2} = -10 \pm 15 \quad \text{and} \quad 10^9 \bar{S}_{29}^{0,2} = -76 \pm 12 \ .$$

These will be used in future, when enough results at different inclinations have accumulated, to determine individual coefficients of order 29. The values of lumped harmonics obtained from analysis of the values of eccentricity were not well defined, because of the high correlations between them and the errors in removing the very large perturbation (31 km) due to odd zonal harmonics.

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